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1990 J. Phys.: Condens. Matter 2 2127

(http://iopscience.iop.org/0953-8984/2/9/002)

J. Phys.: Condens. Matter 2 (1990) 2127–2132. Printed in the UK

Radial stress distribution generated during rapid solidification of amorphous wires

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Received 17 August 1989

Abstract. The magnetic behaviour of highly magnetostrictive amorphous wires is a direct consequence of the residual stresses developed during the sample fabrication. A phenomenological model that assumes a heat transference from a metallic liquid to the water, with cylindrical symmetry, and a solidification process that propagates in the opposite direction to the heat flow has been proposed. The value for the tensile radial stress field has then been calculated. A maximum value for these radial stresses has been found at a distance of about 0.7 R from the axis of the wire.

1. Introduction

The fabrication of amorphous alloy wires with a circular cross section was strongly desired in order to improve the capability of the magnetic and mechanical properties of amorphous metallic ribbons. It was in 1980 that Masumoto's group succeeded for the first time in producing amorphous wires by melt spinning in rotating water [1–3]. The details of the technique have been published elsewhere [2]. The metallic liquid jet transfers heat to the water and, if the jet diameter remains below 130 μm, cooling rates of about 10^6 K s^{-1} can be reached. As a consequence of such a high cooling rate, a strong radial gradient of temperatures develops along the cross section of the jet during the solidification process. Actually, solidification is expected to start in the outer circular shell and then to proceed toward the axis. Propagation of solidification along the inner radial direction gives rise to residual stresses which are frozen in the wire.

Residual stress in ferromagnetic 3d-based metallic glasses is the main source of magnetic anisotropy via magnetoelastic coupling. Therefore the magnetic properties, domain structure and magnetisation processes of amorphous wires are governed by the residual stress distribution. The more peculiar characteristics of the magnetic behaviour of amorphous wires, such as re-entrant magnetic flux reversal with a large Barkhausen discontinuity, are direct consequences of the residual stresses developed during the sample fabrication method [4]. It has been shown that the magnetisation processes as well as their stress dependence can be explained by assuming a normal distribution of residual stress strengths [5]. However, up to the present, no attempt to find the actual stress distribution has been carried out.

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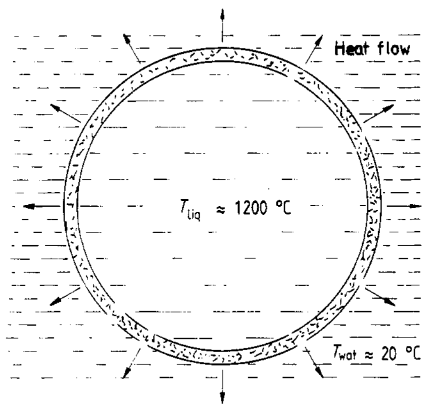

Figure 1. Solidification of the first external shell of the wire. No radial stresses are generated now because the inner melt can flow in the longitudinal direction.

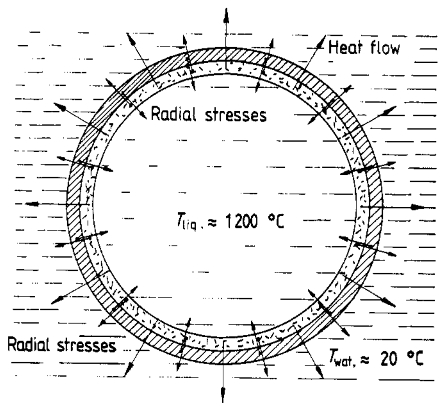

Figure 2. The solidification of the second shell develops the first radial tensile stresses when it solidifies and when it contracts.

The aim of this report is to show a simple phenomenological model that leads us to the more important aspects of the radial dependence of the residual stress.

2. Phenomenological considerations

The calculation outlined here is based upon the following assumptions:

(i) the cylindrical shape of the liquid jet is conserved during the cooling and solidification process;
(ii) the heat is transferred from the metallic liquid to the water according to the same cylindrical symmetry, i.e. lines of heat flow are along the radial directions;
(iii) the outer shell in contact with water solidifies first. Solidification propagates with cylindrical symmetry in the opposite direction to the heat flow;
(iv) the non-simultaneous solidification of the different shells is the source of the residual stress.

The different steps involved in the solidification process of the wire will be as follows: when the molten metal stream hits the water, the outside shell solidifies first establishing the diameter of the wire, $2R$. This shell shrinks but, as it is in outside contact with liquid water and inside contact with a molten core, only negligible radial stress develops (figure 1). Different conditions are present when the next inner shell solidifies and so it shrinks to create the radial stress in the outside shell, which produces a corresponding reaction in the inner shell (see figure 2). Successive solidification processes for the inner shells generate new radial stresses.

3. Calculation of the radial stress

In order to perform the calculation, let us assume the thickness of each successively solidified shell to be r_0. At a given step, n, of the solidification processes there should be

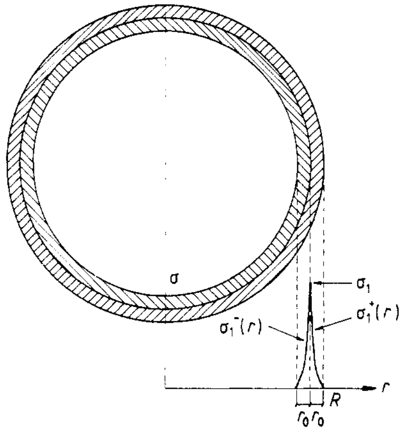

Figure 3. The value of the radial stress, arbitrary units, from the solidification of the second shell must be zero at the limits of the solid part in the wire.

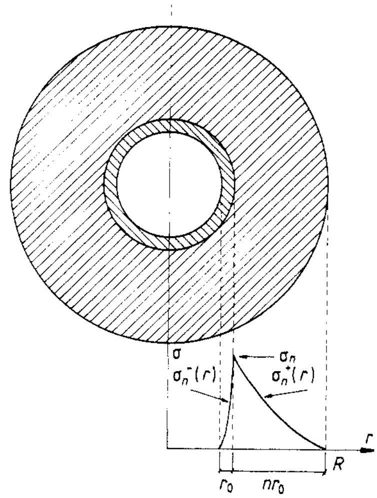

Figure 4. New independent radial stresses are generated by the successive solidification of the different shells.

a solid tube with outer and inner radii, R and $R - nr_0$, respectively surrounded by liquid water and containing molten metal. According to the equilibrium conditions given by the elemental elasticity theory of solid tubes, the radial stress field can be written as [6]

$$\sigma_{rr} = A + B/r^2. \tag{1}$$

The boundary conditions can be written as $\sigma_{rr} = 0$ for $r = R$ and also for $r = R - nr_0$.

The first shell of thickness r_0 is solidified, then the second shell solidifies and on shrinking produces, according to (1) and the boundary conditions, the following tensile stress on the first shell (figure 3):

$$\sigma_1 + (r) = \sigma_1[(R - r_0)^2/(2R - r_0)r_0](-1 + R^2/r^2) \tag{2}$$

for $R - r_0 < r < R$.

The reaction exerted by the first shell on the second one should be

$$\sigma_1 - (r) = -\sigma_1[(R - r_0)^2/(2R - 3r_0)r_0][-1 + (R - 2r_0)^2/r^2] \tag{3}$$

(also a tensile stress and for $R - 2r_0 < r < R - r_0$).

σ_1 is the undetermined parameter which depends on the temperature difference between shells one and two and the thermal conductivity of the alloy. When the third shell solidifies, it shrinks and exerts stress on the outer shell which now has a thickness $2r_0$ and so on. It is easily shown that the stress developed by solidification of the nth shell, figure 4, is given by

$$\sigma_n + (r) = \sigma_n[(R - nr_0)^2/(2R - nr_0)nr_0](-1 + R^2/r^2) \tag{4}$$

for $R - nr_0 < r < R$, and

$$\sigma_n - (r) = -\sigma_n\{(R - nr_0)^2/[2R - (2n + 1)r_0]r_0\}\{-1 + [R - (n + 1)r_0]^2/r^2\} \tag{5}$$

for $R - (n + 1)r_0 < r < R - nr_0$.

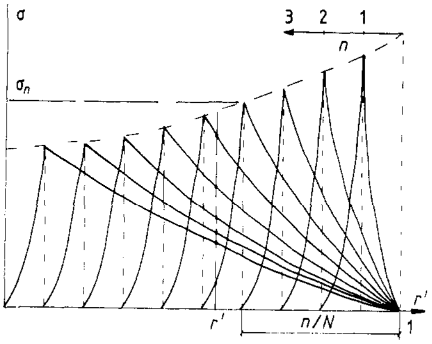

Figure 5. If the solidification in the wire is complete, the value of the total radial stress at each point in the wire will be the superposition of all the stresses generated by each shell.

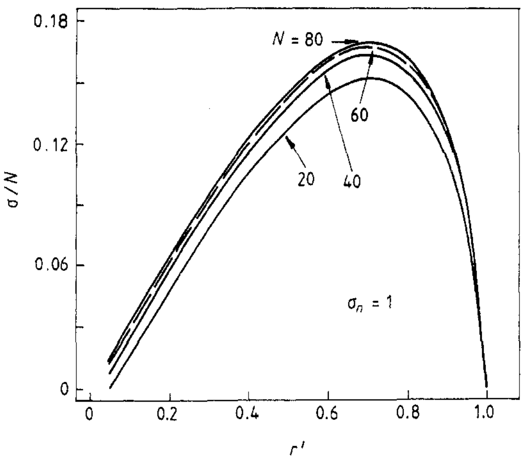

Figure 6. Value of the final stress at each point in the wire when we consider the solidification process with different numbers of shells.

When the wire becomes completely solidified the final stress at each point will be obtained by superposing the stress fields induced by solidification of each shell.

By introducing the new variables $r' = r/R$ $(0 < r' < 1)$ and $N = R/r_0$, the stress at a point located at a distance r' from the wire axis becomes (figure 5)

$$\sigma_n(r') = \sigma_n \frac{(N-n)^2}{2(N-n)-1}\left(1 - \frac{(N-n-1)^2}{N^2 r'^2}\right) + (1 + 1/r'^2) \sum \sigma_n \frac{(N-n)^2}{(2N-n)n}. \qquad (6)$$

The first term on the right hand side corresponds to $n < N(1 - r')$ and the summation of the second term extends from $n > N(1 - r')$ up to N. To calculate this expression, values of N and σ_n should be fixed.

Figure 6 shows the σ_{rr} profiles obtained from equation (6) for different N-values. σ_n was considered to take the value 1 for all n. It is worth noting that the stress profile exhibits a tendency to converge in a unique curve as N increases. From $N = 100$ and for increasing N-values the curve does not undergo relevant changes. This feature suggests the possibility of establishing a continuous formalism for the model, which is now outlined.

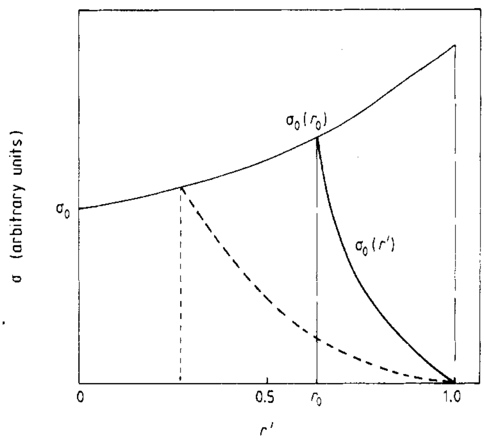

Figure 7. Value of the radial stress generated by the solidification of a shell with infinitesimal thickness and located at r_0.

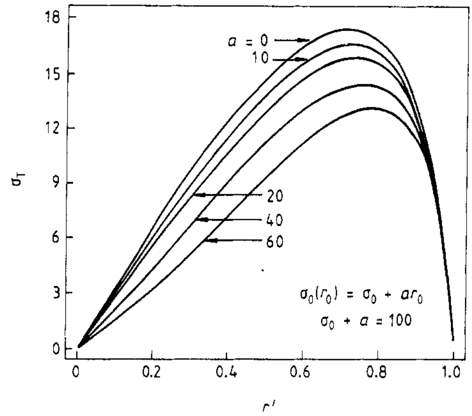

Figure 8. Tensile radial stress field in a wire. The parameter a relates the different cooling rates inside the wire for a constant rate at the surface.

4. Residual stresses from a continuous formalism

Assuming that the main contribution to the total stress at each point comes from the σ^+-terms, one can neglect the σ^--terms. Equation (6) shows that the contribution of the shell placed at $r = r_0$ to the total stress at r' can be written as (figure 7)

$$\sigma_0(r') = \sigma_0 r_0^2(1 - r'^2)/(1 - r_0^2)r'^2. \qquad (7)$$

Thereby, the total stress at $r = r'$ becomes

$$\sigma_{rr}(r') = \frac{1 - r'^2}{r'^2} \int_0^{r'} \sigma(r_0) \frac{r_0^2}{1 - r_0^2} \, \mathrm{d}r_0 \qquad (8)$$

on assuming

$$\sigma(r_0) = \sigma_0 + ar_0 \qquad (9)$$

$$\sigma_{rr}(r') = \frac{1 - r'^2}{r'^2}\left[-r'(\sigma_0 + ar'/2) + \tfrac{1}{2}\ln\left(\frac{1 + r'}{1 - r'}\right) - \tfrac{1}{2}a\ln(1 - r'^2)\right]. \qquad (10)$$

Figure 8 illustrates the stress field profile described by equation (10) for different values of the parameter a (equation (9)). The value of the stress at the shell closer to the external surface of the wire has been kept constant at $\sigma_0 + a = 100$. This assumption means that the cooling rate of the first shell is constant. The value of a describes the rate at which cooling decreases its speed in the inner shells.

5. Conclusions

It has been calculated that the profile of the stress arrangement frozen in amorphous wires exhibits a maximum at 0.7 R. The location of such a maximum would explain the well known value of the magnetic remanence in magnetostrictive wires, which is around

$(0.7)^2M_s$. Also it accounts for the external region, surrounding the inner nucleus, which has a radial distribution of magnetisation.

Further micromagnetic analysis, based on the anisotropy distribution associated with the stress field calculated here, will be carried out.

Acknowledgments

This work was supported, in part, by the Spanish Government (CICYT-PA840365) and also by the USA and Spanish Governments (CCA 8411006).

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